École Doctorale Cerveau-Cognition Comportement
Doctorate in Theoretical Neuroscience
CAMS (CNRS-EHESS)
Doctorate in Mathematics
Department of Mathematics (University of Bologna)

Marta Favali

Thesis director: Alessandro Sarti
Thesis co-director: Giovanna Citti

Title of the project: Formal models of visual perception based on cortical architectures.

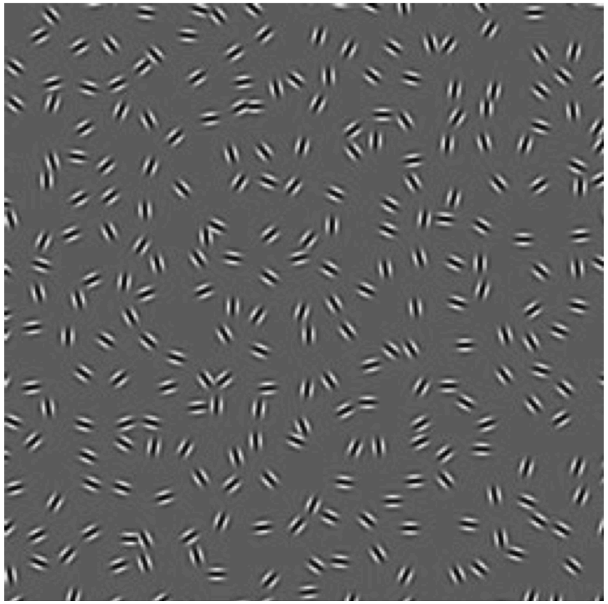
Objectives

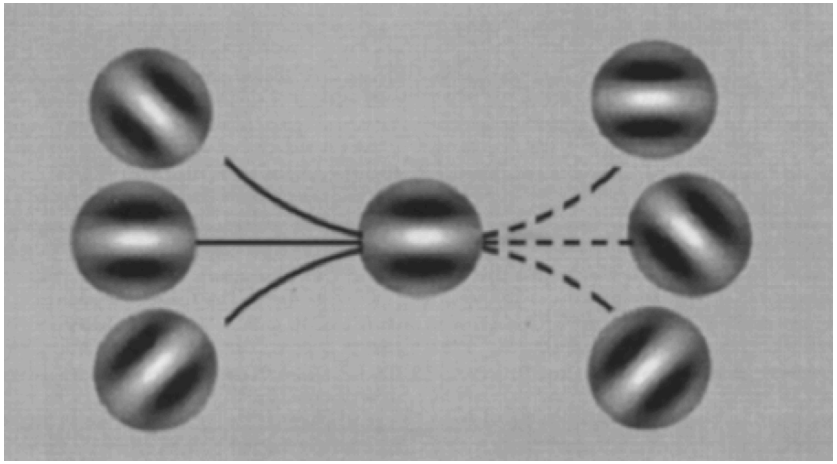
- Mathematical models of the primary visual cortex
- Mathematical models of visual perception

Methods and development of work:

- The neurogeometry of the visual cortex
- Models of cortical connectivity, with different stochastic kernels
- Spectral analysis of connectivity matrix
- Simulations (Kanizsa figures and retinal images).

Individuation of perceptual units: the association fields





Field et al, 1993

Mathematical models of the functional architecture of V1

- J.J. Koenderink, A.J van Doorn, *Representation of local geometry in the visual system.*, Biol. Cybernet. 55,367-375, 1987.

- J. Petitot, *The neurogeometry of pinwheels as a sub-Riemannian contact structure*, in Journal Physiol, Pages 97(2-3):265-309, 2003.

- G. Citti, A.Sarti, *A cortical based model of perceptual completion in the roto-translation space*, Journal of Mathematical Imaging and Vision, 24(3):307-326, 2006

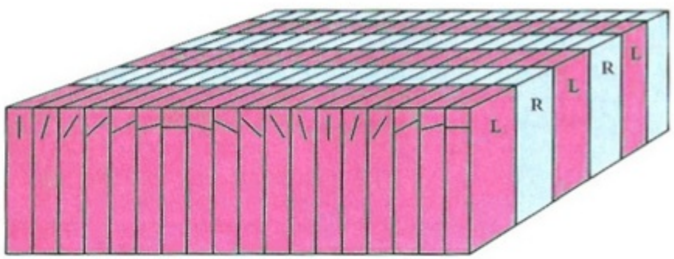
- S.W. Zucker, *Differential geometry from the Frenet point of view: boundary detection, stereo, texture and color.*, In: Paragios, N., Chen, Y., Faugeras, O. (eds.) Handbook of Mathematical Models in Computer Vision, pp. 357-373. Springer, US, 2006.

- A.Sarti, G. Citti, J. Petitot, *The symplectic structure of the primary visual cortex*, Biol. Cybern. 98, 33-48, 2008.

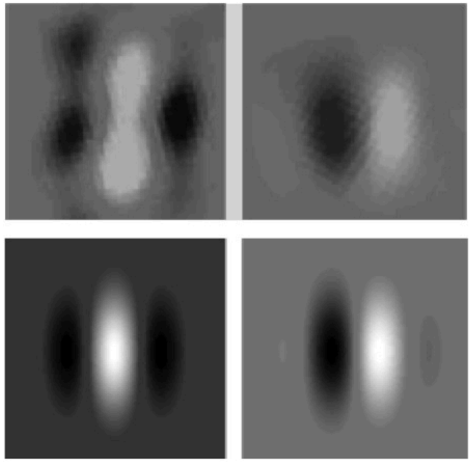
- R. Duits, E.M. Franken, *Left invariant parabolic evolution equations on SE(2) and contour enhance- ment via invertible orientation scores, part I: Linear left-invariant diffusion equations on SE(2)*, Q. Appl. Math. 68, 255-292, 2010.

The neurogeometry of V1

Hypercolumnar structure



Hubel-Wiesel, 1965



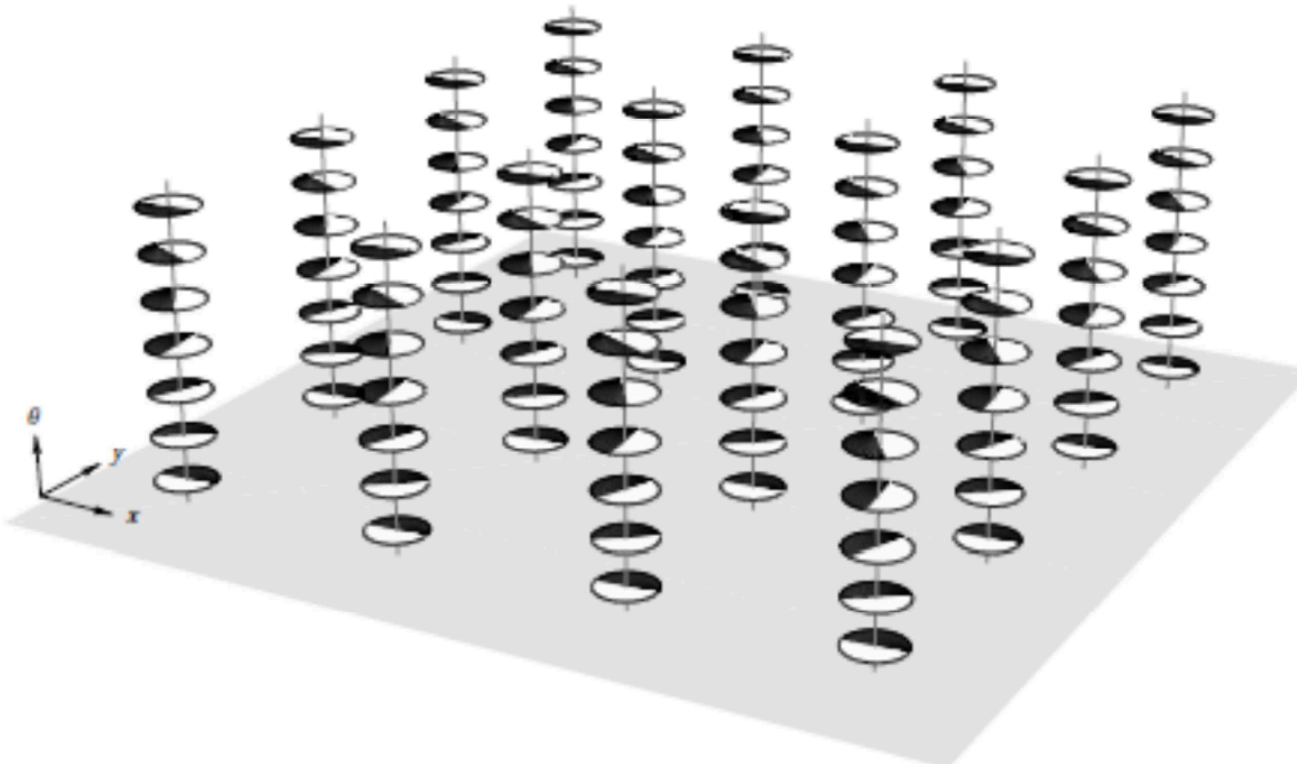
Receptive profile of a simple cell and its representation as a even-symmetric and odd-symmetric Gabor filters.

$$\varphi(x,y,\theta)=\frac{1}{2\pi\sigma^2}e^{[-\frac{(\tilde{x}^2+\tilde{y}^2)}{\sigma^2}+i\frac{\tilde{y}}{\sigma}]}$$

$$\tilde{x}=x\cos(\theta)+y\sin(\theta)$$

$$\tilde{y}=-x\sin(\theta)+y\cos(\theta)$$

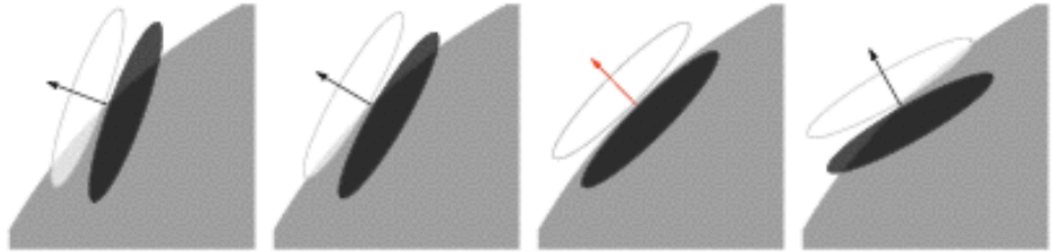
Daugman, 1985



- Simple cells are modeled with Gabor filters and represent a group:

$$G = \begin{pmatrix} \cos(\theta) & -sin(\theta) & x \\ sin(\theta) & cos(\theta) & y \\ 0 & 0 & 1 \end{pmatrix}$$

$\vec{X}_1 = (\cos\theta, \sin\theta, 0)$ $\qquad$ $\vec{X}_2 = (0,0,1)$ $\qquad$ $\vec{X}_3 = (-\sin\theta, \cos\theta, 0)$

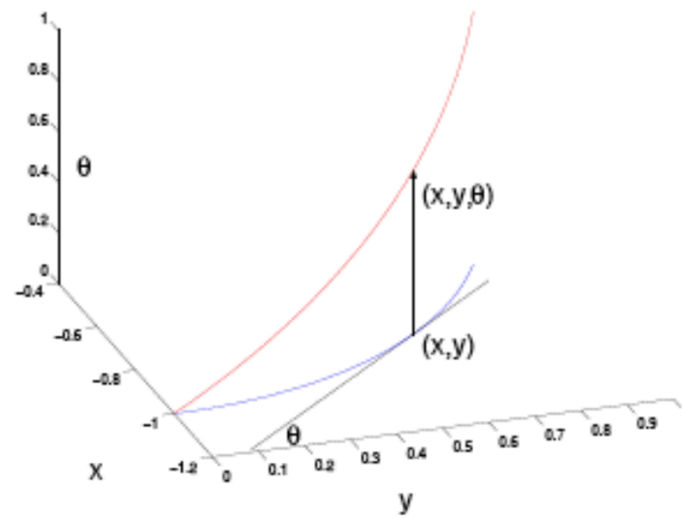


Output of simple cells:

$$h(x,y,\theta)=\int\varphi_{x,y,\theta}(x',y')I(x',y')dx'\,dy'$$

Lifting: nonmaximal suppression

$$max_{\theta}(h(x,y,\theta))=h(x,y,\bar{\theta})$$



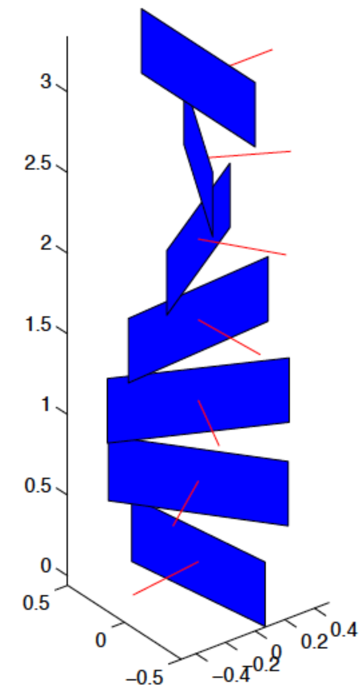
Sarti Citti, 2006

$$\vec{X}_1 = (\cos\theta, \sin\theta, 0) \qquad \vec{X}_2 = (0,0,1)$$

$$X_1 = \cos(\theta)\partial_x + \sin(\theta)\partial_y \qquad X_2 = \partial_\theta$$

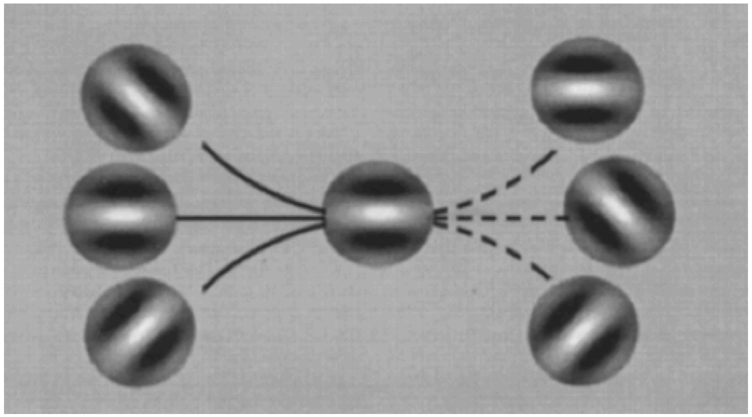
$$X_3 = [X_2, X_1] = -\sin(\theta)\partial_x + \cos(\theta)\partial_y$$

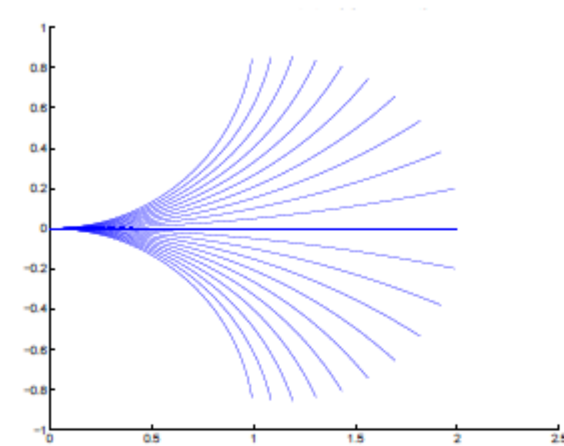
$\vec{X}_1, \vec{X}_2, \vec{X}_3$ generator of the tangent space.



Citti-Sarti, 2006

Differential model of Citti-Sarti





$$X_1 = \cos(\theta)\partial_x + \sin(\theta)\partial_y$$

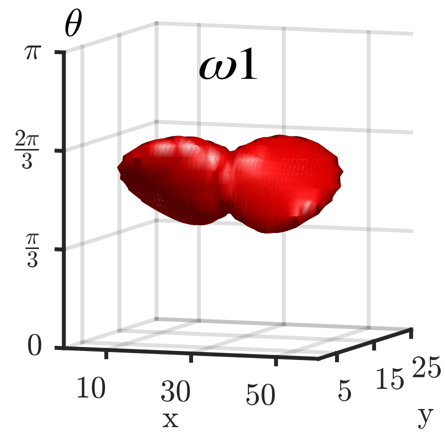
$$X_2 = \partial_\theta$$

$$\begin{cases} \gamma'(t) = \vec{X}_1(\gamma) + k\vec{X}_2(\gamma) \\ \gamma(0) = (x_0, y_0, \theta_0) \end{cases}$$

Citti-Sarti, 2006

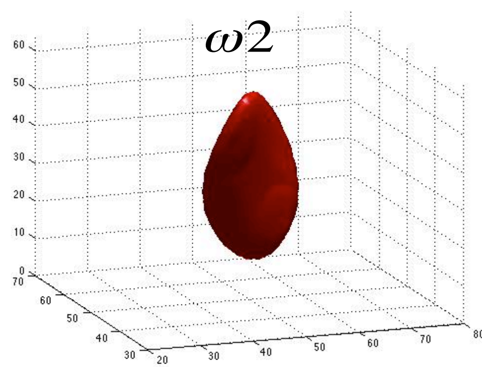
The Fokker Planck operator has a nonnegative fundamental solution Γ_1 that satisfies:

$$X_1\Gamma_1((x,y,\theta),(x',y',\theta'))+\sigma^2 X_{22}\Gamma_1((x,y,\theta),(x',y',\theta'))=\delta(x,y,\theta)$$



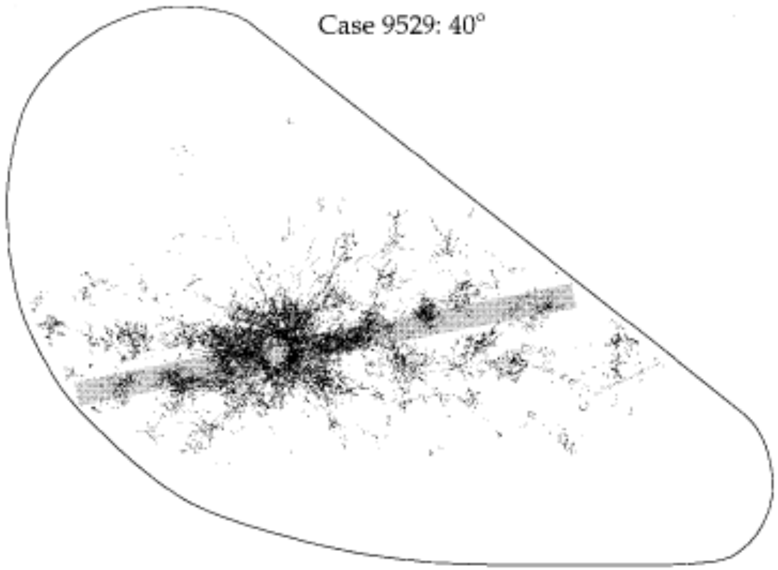
The Sub-Riemannian Laplacian operator has a nonnegative fundamental solution Γ_2 that satisfies:

$$\sigma_1^2 X_{11}\Gamma_2((x,y,\theta),(x',y',\theta'))+\sigma_2^2 X_{22}\Gamma_2((x,y,\theta),(x',y',\theta'))=\delta(x,y,\theta)$$

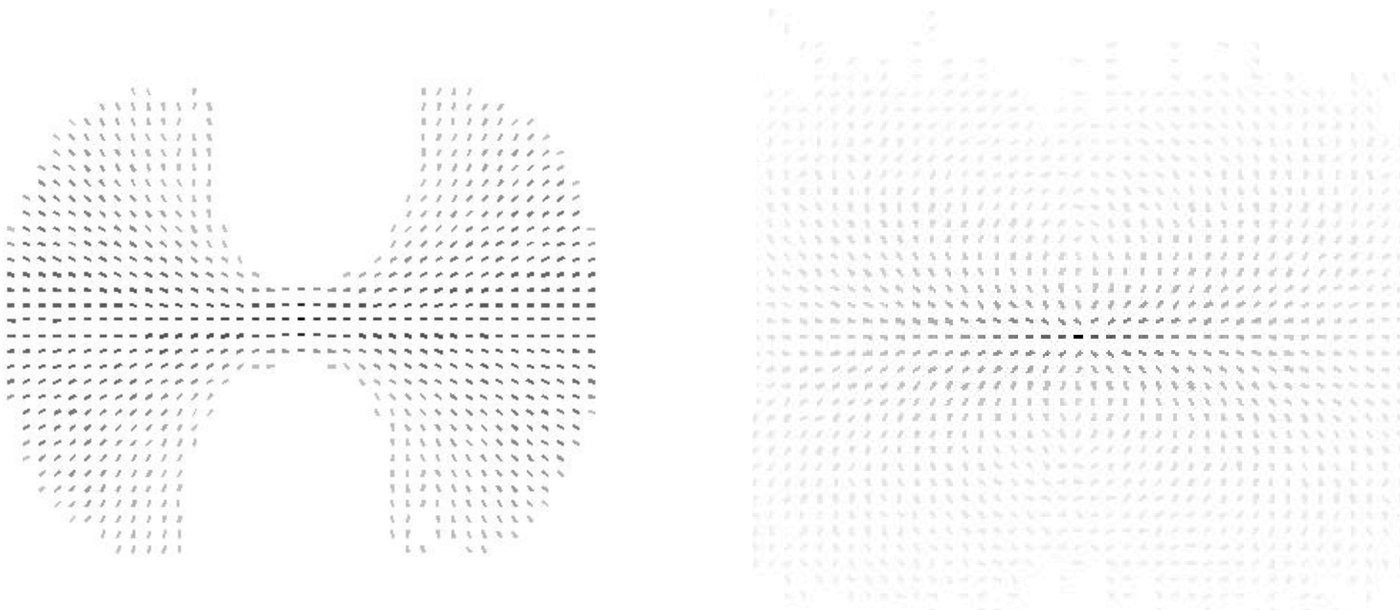


Sanguinetti Citti Sarti, 2008

The connectivity map measured by Bosking in tree shrew:



Bosking et al, 1997



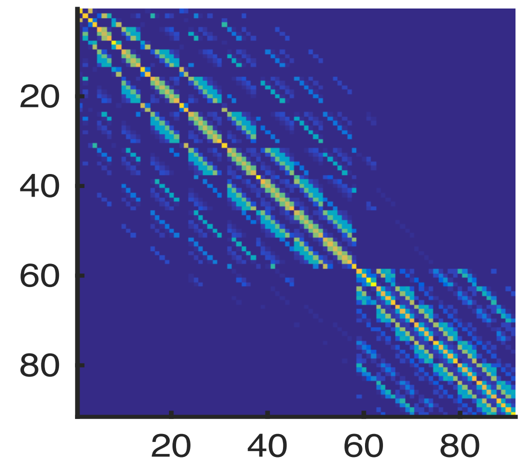
Maximum values along θ dimension of the connectivity kernels associated to the fundamental solution of a FP (left) and SRL equations (right).

Affinity Matrix

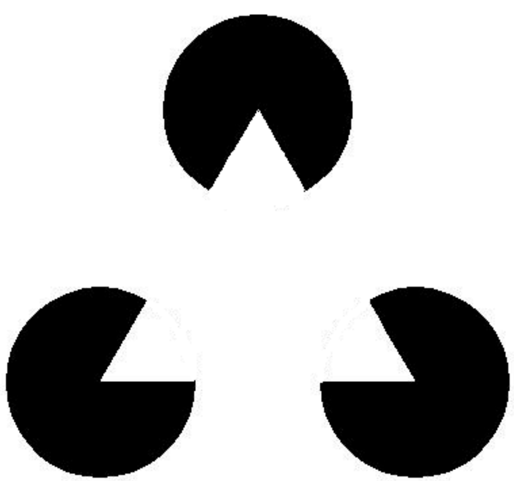
Propagation of $h(x_i, y_i, \overline{\theta}_i)$ to close cells:

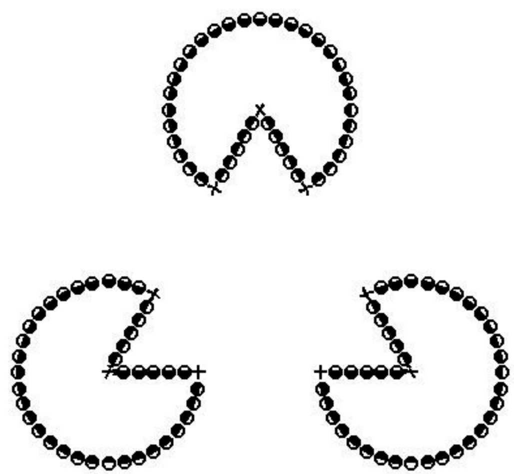
$$\sum_{j=1}^{N} \omega((x_i, y_i, \overline{\theta}_i), (x_j, y_j, \overline{\theta}_j)) h(x_j, y_j, \overline{\theta}_j)$$

$$A_{i,j} = \omega((x_i, y_i, \overline{\theta}_i), (x_j, y_j, \overline{\theta}_j))$$



Individuation of perceptual units: Kanizsa figure



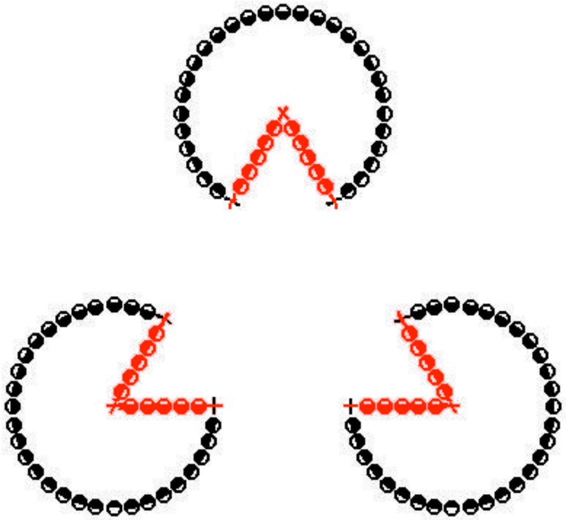


Numerical algorithm

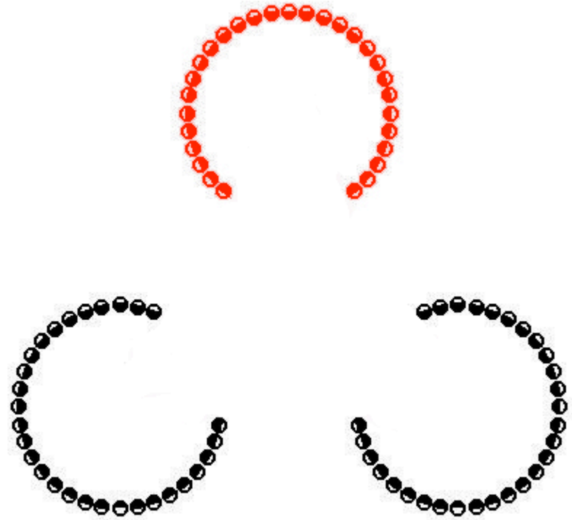
1. Define the affinity matrix $A_{i,j}$ from the approximated connectivity kernel.

2. Solve the eigenvalue problem $A_{i,j}u_i = \lambda_i u_i$, where the order of *i* is such that λ_i is decreasing.

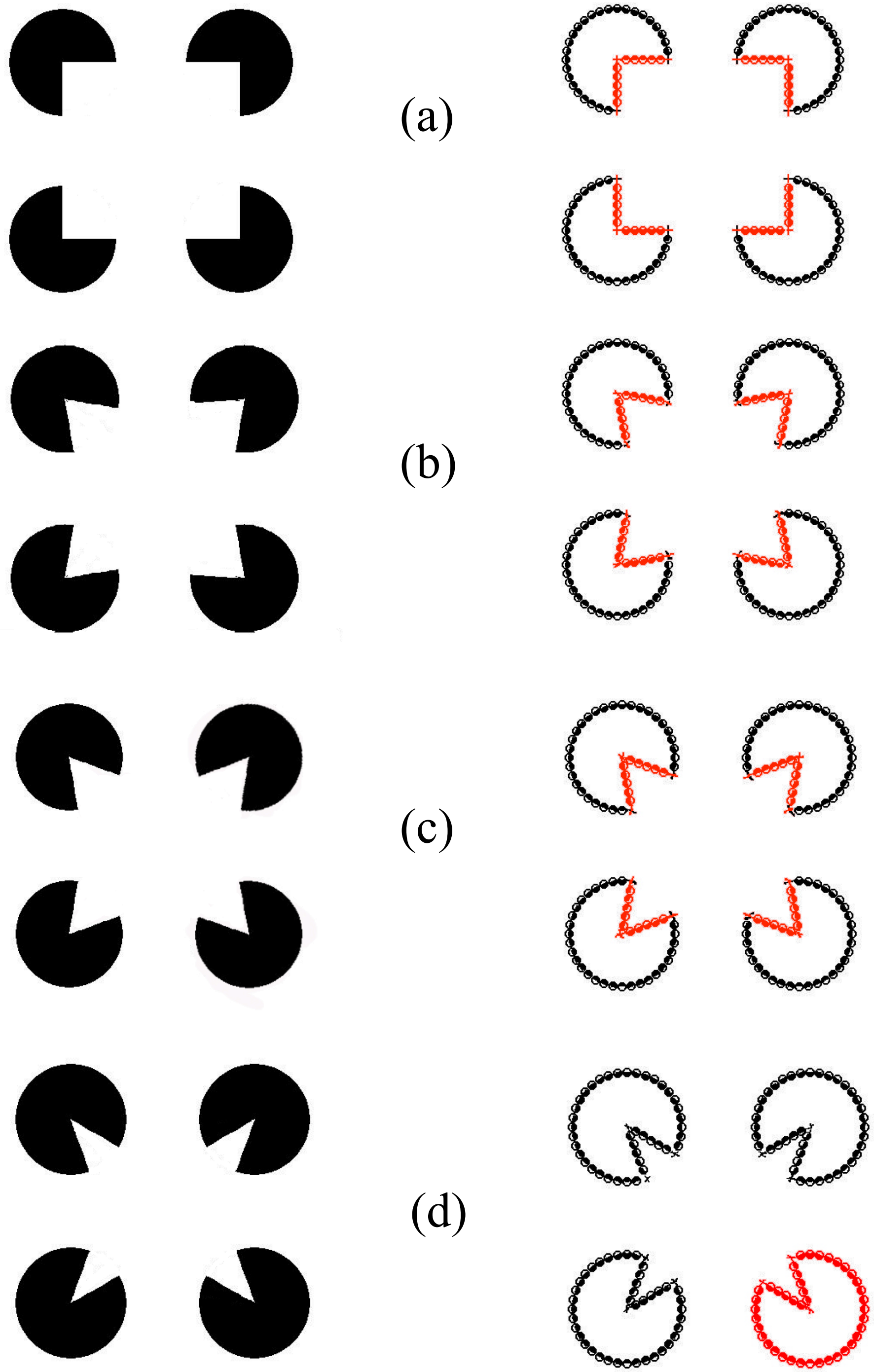
3. Find and represent on the segments the eigenvector u_1 associated to its largest eigenvalue.



First eigenvector of the affinity matrix, using the fundamental solutions of FP and SRL equations.



The affinity matrix is updated removing the detected perceptual unit; the first eigenvector of the new matrix is visualized.



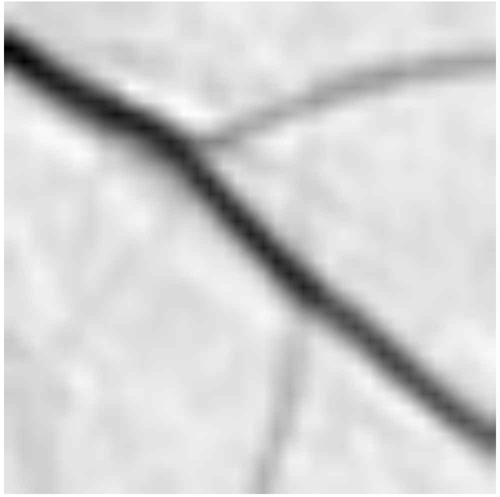
In red the first eigenvectors of the affinity matrix using both connectivity kernel.

F., Citti, Sarti: “Local and global gestalt laws: A neurally based spectral approach”, submitted to Neural Computation, 2015.

Individuation of perceptual units: retinal images

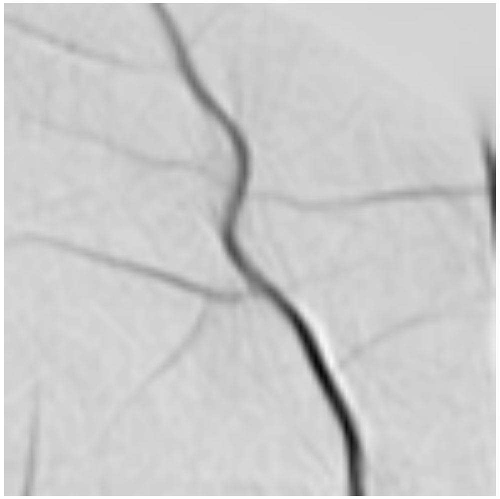
Analyzed problems:

bifurcation



crossing

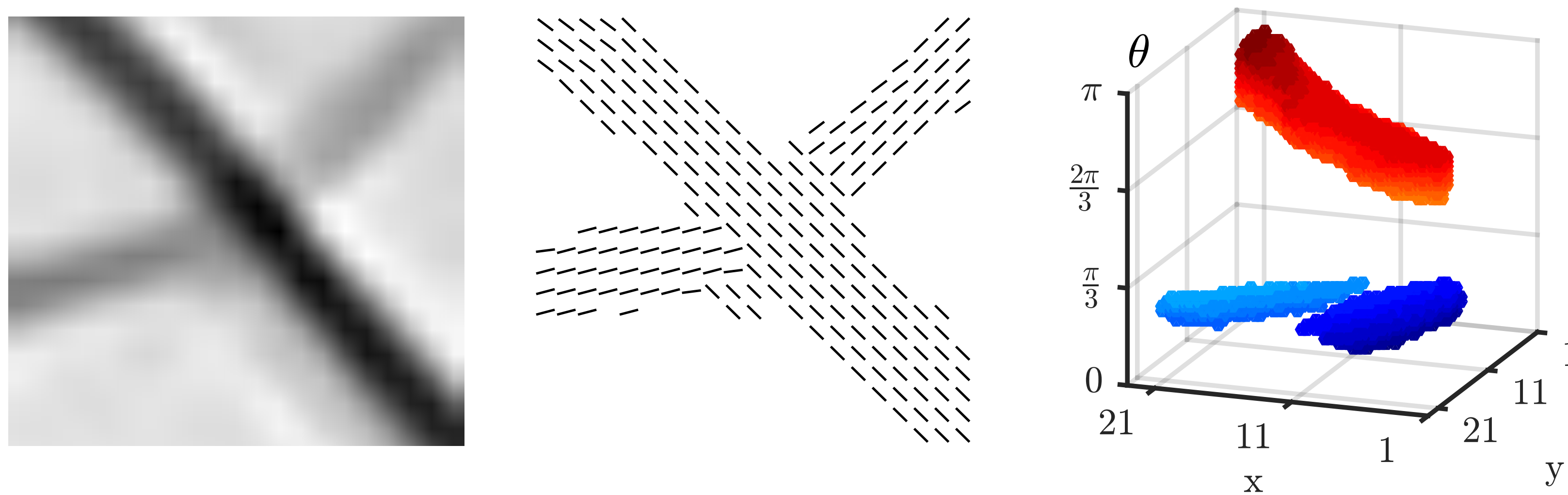
disconnected vessels



In collaboration with TU/e

- In presence of an input stimuli, the visual cortex codifies the features of position and orientation.

Image patch: crossing | Oriented segments | Lifted image



- The proposed method models the connectivity as the fundamental solution of the Fokker-Planck equation.

- In order to measure the distances between intensities we introduce the kernel ω_3 :

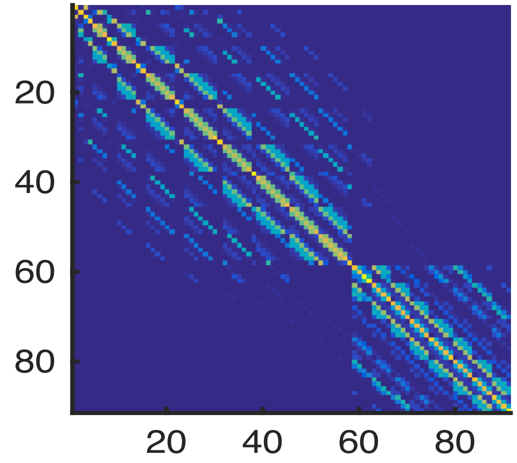
$$\omega_3(f_i, f_j) = e^{(-\frac{1}{2}(\frac{f_i - f_j}{\sigma_2}))^2}$$

- The final connectivity kernel can be written as the product of the two components:

$$\omega\ ((x_i, y_i, \theta_i, f_i), (x_j, y_j, \theta_j, f_j)) = \omega_1((x_i, y_i, \theta_i), (x_j, y_j, \theta_j))\omega_3(f_i, f_j)$$

- Starting from that connectivity kernel it is possible to extract perceptual units from images by means of spectral analysis of suitable affinity matrix:

$$A_{ij} = \omega\ ((x_i, y_i, \theta_i, f_i), (x_j, y_j, \theta_j, f_j))$$



Normalized Spectral Clustering

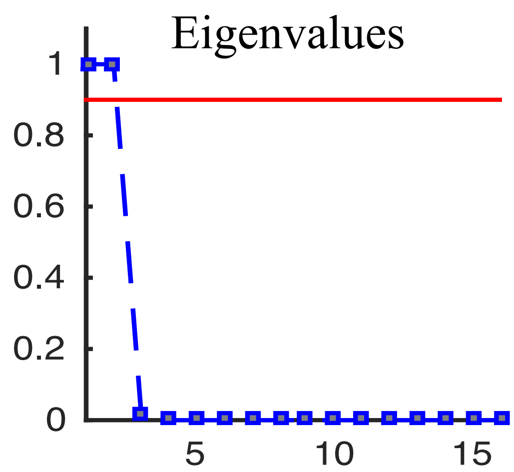
1. After defining the affinity matrix A from the connectivity kernel

2. We evaluate the normalized affinity matrix $P = D^{-1}A$ where D is the diagonal degree matrix having elements:

$$d_i = \sum_{j=1}^{n} a_{i,j}$$

3. Solve the eigenvalue problem: $Pu_m = \lambda_m u_m$

4. Define the thresholds ε, τ and evaluate the largest integer K such that $\lambda_m^{\tau} > 1 - \varepsilon$ for $m = 1, \ldots, K$



Shi Malik, 2000

Meila Shi, 2001

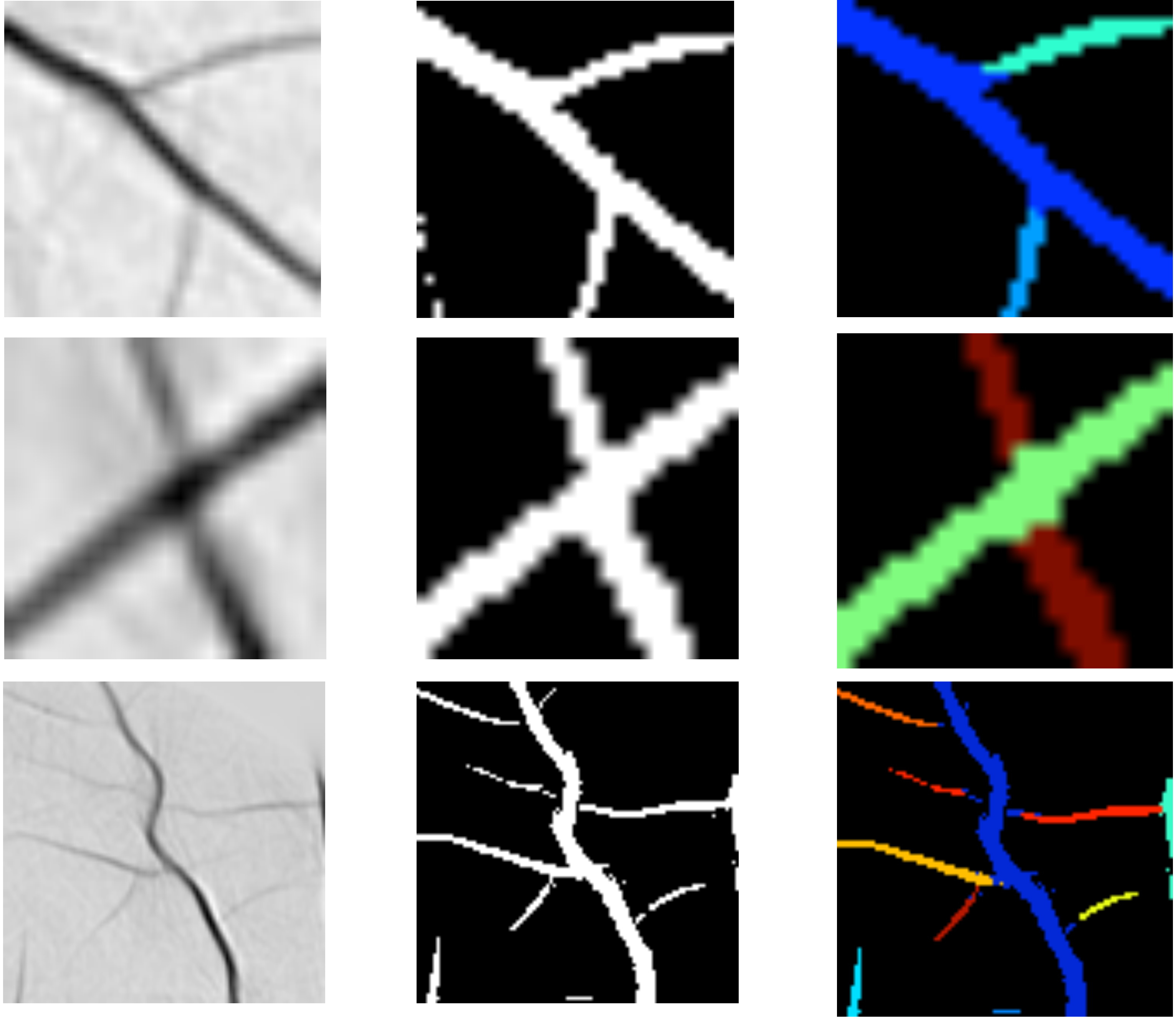
Normalized Spectral Clustering

5. Define the clusters from the u_K eigenvector

6. Find and remove the clusters that contain less than a minimum cluster size elements.

Image patch

Perceptual units



F., Abbasi, Romeny, Sarti: “Analysis of Vessel Connectivities in Retinal Images by Cortically Inspired Spectral Clustering”, submitted to JMIV, 2015.

Conclusion

- We have presented a neurally based model for figure-ground segmentation using spectral methods.
- Different connectivity kernels are compatible with the functional architecture of V1, we have compared their properties and modelled them as fundamental solution of Fokker Planck, Sub-Riemannian Laplacian equations.
- With this model we have identified perceptual units of different Kanizsa figures and retinal images.
- We have shown how this can be considered a good quantitative model for the constitution of perceptual units.

Future work

- Our method represents some limitations at blood vessels with high curvature. These structures will be analyzed in an higher dimensional group (Engel group) adding other features.

- Other images containing tree structures will be analyzed.

- We will compare the results obtained with this model with functional fMRI data, that represent measurements of cortical neural activity.

Thanks for your attention