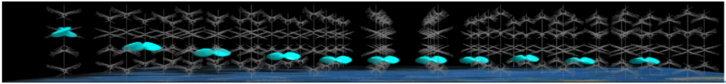

MAnET MID-TERM REVIEW MEETING

Minimal cones and calibrations

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Almgren minimisers

Let us fix $0 < d < n$, E will be a subset of $\mathbb{R}^n$ with locally finite d-dimensional Hausdorff measure $\mathcal{H}^d$.

Admissible competitors

Let $U \subset \mathbb{R}^n$ be open. F is an admissible competitor for E on U if there exists a continuous function $\phi : [0,1] \times U \to U$ such that:

- $\phi(0,\cdot) = Id$;
- $\exists K \subset\subset U$ such that $\forall t \; \phi(t,\cdot) = Id$ in K^c;
- $\phi(1,\cdot)$ is Lipschitz and $F = \phi(1,E)$.

We call ϕ an admissible deformation.

Almgren minimiser

E is an Almgren minimiser if for every admissible competitor F we have:

$$\mathcal{H}^d(E \setminus F) \leq \mathcal{H}^d(F \setminus E).$$

Almgren minimisers

Properties

If E is an Almgren minimiser then:

- E is rectifiable;
- E is Ahlfors regular;
- E is uniformly rectifiable;
- there exists an $\mathcal{H}^d$-negligible $N \subset \mathbb{R}^n$ such that $E \setminus N$ is a $C^{1,\alpha}$ d-dimensional submanifold of $\mathbb{R}^n$;
- $\forall x \in E$, $\forall \mu \in Tan^d(\mathcal{H}^d \llcorner E, x)$ is supported on a minimal cone.

Remark

Let $E \subset \mathbb{R}^n$ be an Almgren minimiser:

- E is an Almgren minimiser as a subset of $\mathbb{R}^m$ with $m > n$ because since projections do not increase the Hausdorff area;
- let $k > 0$, by a slicing argument $E \times \mathbb{R}^k$ is an Almgren minimiser in $\mathbb{R}^{n+k}$.

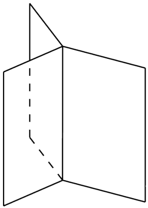
Minimal Cones

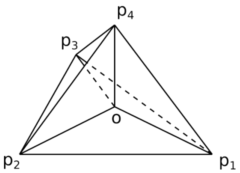
$\mathbb{R}^2$

There are only two types of minimal cones: the straight line; and three half lines meeting with angle of 120°, which we will denote as Y.

$\mathbb{R}^3$

There are no new 1-dimensional cones.
The 2-dimensional minimal cones are of three kinds: planes; $\mathbb{Y} := Y \times \mathbb{R}$; the cone over the edges of a regular tetrahedron $\mathbb{T}$.





Minimal Cones

Higher dimensions

- cone over $sk_{n-2}(Q^n)$ for $n \geq 4$ [Brakke 1991];
- cone over $sk_{n-2}(\Delta_n)$ [Morgan 1994];
- for any $d, m \geq 2$ there exists $\theta(m, d) \in (0, \pi/2)$ such that, given $P_1, \ldots, P_m$ d-planes in $\mathbb{R}^{md}$, their union is an Almgren minimiser if all the characteristic angles are greater than $\theta(m, d)$ [Liang 2013];
- $Y \times Y \subset \mathbb{R}^4$ [Liang 2014].

Sliding Boundary

Sliding deformation

Let $U \subset \mathbb{R}^n$ be open and $\Gamma_i \subset U$, $1 \leq i \leq I$, be a finite collection of closed sets. A sliding deformation with respect to $\{\Gamma_i\}_i$ is a continuous function $\phi : [0,1] \times U \to U$ such that:

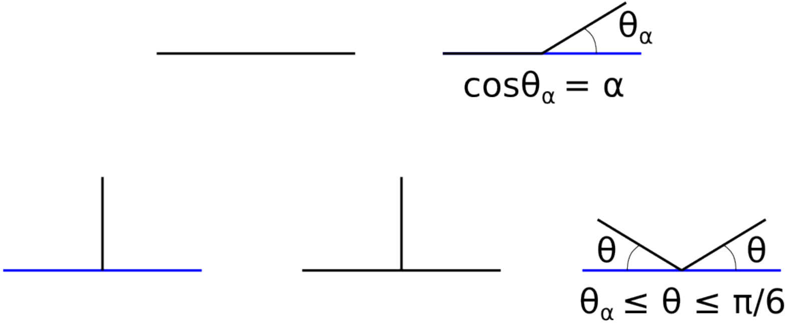
- $\phi(0, \cdot) = Id$;
- $\exists K \subset\subset U$ such that $\forall t$ $\phi(t, \cdot) = Id$ in K^c;
- $\phi(1, \cdot)$ is Lipschitz and $F = \phi(1, E)$;
- $\phi(t, x) \in \Gamma_i$ $\forall t$ if $x \in \Gamma_i$.

Sliding boundary minimisers

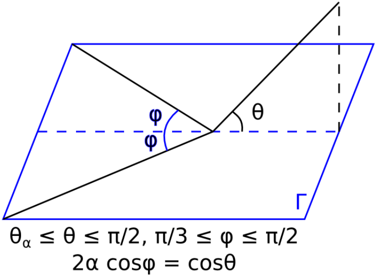
Given $\Gamma := \cup_i \Gamma_i$ and $0 \leq \alpha \leq 1$ we define a new cost functional $c(E) := \mathcal{H}^d(E \setminus \Gamma) + \alpha \mathcal{H}^d(E \cap \Gamma)$ and we say that E is a sliding boundary minimiser if for any competitor F obtained as image of a sliding deformation we have $c(E \setminus F) \leq c(F \setminus E)$.

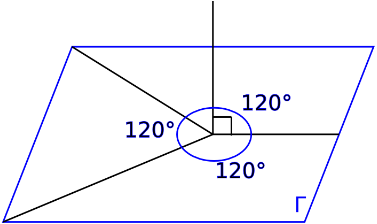
Sliding minimal cones

One-dimensional minimal cones in $\mathbb{R}^2_+$, the sliding boundary $\Gamma := \{y = 0\}$ is in blue.



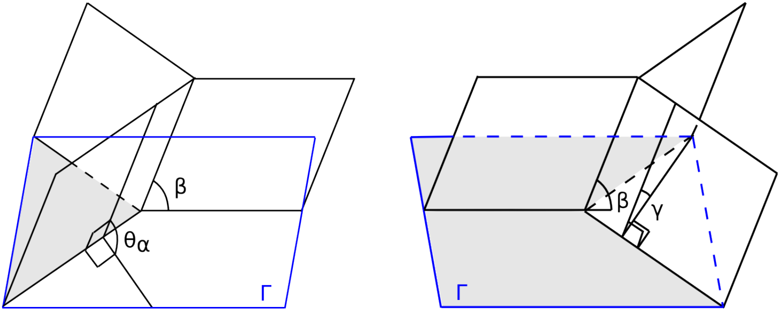
One-dimensional minimal cones in $\mathbb{R}^3_+$, the sliding boundary $\Gamma := \{z = 0\}$ is in blue.





Sliding minimal cones

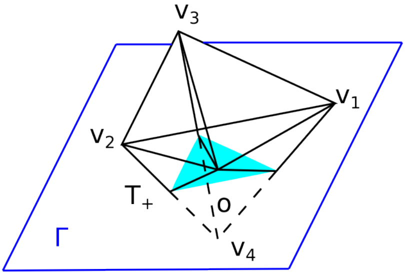
Two-dimensional cones in $\mathbb{R}^3_+$, the sliding boundary $\Gamma := \{z = 0\}$ is in blue, the intersection between Γ and the cone is in grey.



The left cone, which we call $\mathbb{Y}_\beta$, is minimal if $\cos\beta = \alpha\frac{2}{\sqrt{3}}$, the right cone is not minimal.

Sliding minimal cones

Let us set $\mathbb{T}^2_+$ as the cone over $sk_1(\Delta_3) \cap \mathbb{R}^3_+$, then $\mathbb{T}_+$ is a sliding minimal cone if $\alpha \geq \sqrt{\frac{2}{3}}$.



In general, let $\mathbb{T}^{n-1}_+$ be the cone over $sk_{n-1}(\Delta_n) \cap \mathbb{R}^n_+$, then $\mathbb{T}_+$ is a sliding minimal cone if $\alpha \geq \frac{1}{\sqrt{2}}\sqrt{\frac{n+1}{n}}$.

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THANK YOU FOR YOUR ATTENTION