Gauss curvature in the Heisenberg group: a proposal

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Abstract

The notion of Gauss and mean curvature play a crucial role in the study of differential geometry of smooth Euclidean surfaces embedded in $\mathbb{R}^3$.

In the first Heisenberg group $\mathbb{H}$ there is a currently well accepted notion of *horizontal mean curvature* for smooth Euclidean surfaces $\Sigma \subset \mathbb{H}$, but it is still not understood what could be a reasonable candidate for the notion of *horizontal Gauss curvature*. In this talk I will suggest a possible definition based on the so called Riemannian approximation scheme. If time permits, I will also present a Heisenberg version of the well known Gauss-Bonnet theorem for Euclidean C^2-smooth, oriented and compact surfaces in $\mathbb{H}$.

These results are connected to the formulation of a Steiner-type formula in $\mathbb{H}$ already developed under the supervision of Prof. Zoltan Balogh.

One of the future goal of my project supervised by Prof. Bruno Franchi, is to generalize these results and to relate them to the Rumin intrinsic complex of differential forms.