1 Formal models of visual perception based on cortical architectures

In presence of an input stimuli, the visual cortex codifies the features of position and orientation. Considering the geometry of the cortex generated by the vector fields X_1 and X_2 defined in [2]:

$$\vec{X}_1 = (\cos\theta, \sin\theta, 0), \quad \vec{X}_2 = (0, 0, 1), \tag{1}$$

we can model the cortical connectivity with different stochastic kernels. These kernels are fundamental solutions of suitable operators, functions of the vectors X_1 and X_2.
If we describe the long range propagation with a deterministic component in direction X_1 (which describes the long range connectivity) and stochastic component along X_2 (the direction of intracolumnar connectivity) we obtain that the associated probability density is the fundamental solution Γ_1 of Fokker Planck equation, that satisfies:

$$X_1\Gamma_1((x,y,\theta),(x',y',\theta')) + \sigma^2 X_{22}\Gamma_1((x,y,\theta),(x',y',\theta')) = \delta(x,y,\theta). \tag{2}$$

If we assume that intracolumnar and long range connections have comparable strength, we have to modify the equation of long range propagation, since the coefficients of propagation in both directions X_1 and X_2 are stochastics. The operator reduces to the Sub-Riemannian Laplacian whose fundamental solution Γ_2 satisfies::

$$X_{11}\Gamma_2((x,y,\theta),(x',y',\theta')) + \sigma^2 X_{22}\Gamma_2((x,y,\theta),(x',y',\theta')) = \delta(x,y,\theta). \tag{3}$$

Both these kernels are in good agreement with the connectivity measured by Bosking in tree shrew [1].
Starting from these kernels, the problem of grouping has been faced by means of spectral analysis of suitable affinity matrices [3,4]. At every point (x_i, y_i) with gradient sufficiently large w call θ_i the direction of the gradient, and obtain a point $(x_i, y_i, \theta_i$ of the group. We denote ω the symmetrization of Γ, we call affinity matrix the kernel restricted to the selected points:

$$A \in R_{nxn} \quad where \quad A_{i,j} = \omega((x_i, y_i, \theta_i), (x_j, y_j, \theta_j)) \quad with \quad i,j = 1, ..., n \tag{4}$$

The eigenvectors u_m of the affinity matrix represent the perceptual units and the salient objects in the scene corresponds to eigenvectors with highest eigenvalues. In [4] it has been observed that this spectral analysis can be produced by the neural activity in the primary visual cortex, by means of symmetry breaking of solutions of mean field equation. For the numerical simulations, I have considered particularly Kanizsa figures as clear examples of problems of visual perception.
The model of cortical connectivity obtained as fundamental solution of the Fokker Planck equation has been applied to the analysis of retinal images, to afford problems of grouping during the tracking of blood vessels. Combining spectral analysis and spectral clustering algorithm [5] it has been possible to group different perceptual units in these images.

2 Bibliography

[1] W. Bosking,Y. Zhang, B. Schofield, D. Fitzpatrick, *Orientation selectivity and the arrangement of horizontal connections in tree shrew striate cortex*, J. Neurosi, pag 2112-2117, 1997.

[2] G. Citti and A. Sarti,*A cortical based model of perceptual completion in the roto-translation space*, Journal of Mathematical Imaging and Vision, 24(3):307-326, 2006.

[3] A. Sarti, G. Citti, *On the origin and nature of neurogeometry*, La Nuova Critica, 2011.

[4] A. Sarti, G. Citti, *The constitution of visual perceptual units in the functional architecture of V1*, Journal of computational neuroscience, to appear, 2014.

[5] U. Von Luxburg,*A tutorial on spectral clustering*, Statistics and computing 17(4), 395416 (2007).