Minimal cones and calibrations

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The Plateau problem arises from physics, and in particular from soap bubbles and soap films. Solving the Plateau problem means to find the surface with minimal area among all the surfaces with a given boundary. Part of the problem actually consists in giving a suitable definition to the notions of "surface", "area" and "boundary". Given $0 < d < n$ we will consider a setting, due to Almgren, in which the considered objects are sets with locally finite d-dimensional Hausdorff measure, the functional we will try to minimise is the Hausdorff area $\mathcal{H}^d$, and the boundary condition is given in terms of a one-parameter family of deformations. Almgren minimisers turn out to have nice regularity properties, in particular an Almgren minimiser is a $C^{1,\alpha}$ embedded submanifold of $\mathbb{R}^n$ up to a negligible set, and the tangent cone to any point of such a minimiser is a minimal cone. Therefore in order to give a complete characterisation of these object we need to know how minimal cones look like. The complete list of minimal cones of $\mathbb{R}^2$ and $\mathbb{R}^3$ is well known since long time while in higher dimensions the list if far from being complete and we only know few examples. I will also talk about a small variation of this setting which we call "sliding boundary".